

Modifying a Board Game to Foster Kindergartners' Logico-Mathematical Thinking

Constance Kamii

Do you remember playing the game Sorry! in your childhood? When I was looking for games to support first grade arithmetic learning, I thought about using this board game by Parker Brothers. However, Sorry! seemed too easy for first grade arithmetic because it does not involve any addition.

To play Sorry! each player moves four markers toward his or her own goal. Players take turns advancing their markers according to the instructions printed on the cards they turn over. Most of the cards tell players to move forward 3, 5, 8, 10, or 12 squares, but the Sorry! card says, "Must take one man from your Start, place it on any square that is occupied by any opponent, and return that opponent's man to Start." The winner is the first person to place all four markers in his or her goal.

To give first-graders opportunities to learn addition, I replaced the cards with a 10-sided die and changed the rule to "Roll the die and advance by double the number rolled." If a 6 came up, for example, the player would take 12 steps, and if a 4 came up, he or she would take 8 steps (Kamii 1985). When first-graders play math games like this day after day, they learn arithmetic without any worksheets.

A group of 12 early childhood educators teaching five- and six-year-olds in Japan decided to further

modify Sorry! because the first grade version was too hard to encourage kindergartners' thinking. The teachers are part of a large group formed in the 1980s in five Japanese cities to study Piaget's theory. These teachers base their practice on Piaget's constructivism as discussed by Kamii and DeVries (1977, 1980, [1978] 1993) and DeVries and Zan (1994).

According to Piaget's constructivism, children develop their ability to think logico-mathematically by reasoning actively. (Logico-mathematical knowledge will be explained shortly.) Constructivist teachers therefore look for every opportunity throughout the day to encourage children to reason. When a child spills milk, for example, a teacher is likely to ask, "What do you need to do?" or "Would you like me to help you clean it up?" depending on the child's ability to make an appropriate decision in such a situation.

This article describes the modifications the 12 teachers made to Sorry! so that other teachers might choose or modify other games to encourage children to think logico-mathematically. The article first describes the final version of the

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Twelve teachers in Fukuyama City, Japan, collaborated to modify Sorry! for kindergartners: Yukie Fujii, Hiromi Iwamoto, Sachiko Kohzuka, Masako Konishi, Yumi Matsumoto, Kazumi Mihira, Fumi Miyachi, Chiharu Nawachi, Shiho Saragai, Mitsue Shimizu, Ikuyo Takeuchi, and Mitsuyo Yanagida.

kindergarten board game and follows with an explanation of why the changes were made. The teachers made the modifications in several stages by changing part of the game, observing how children played, then making further changes and testing them in classrooms.

The kindergarten game and the rationale

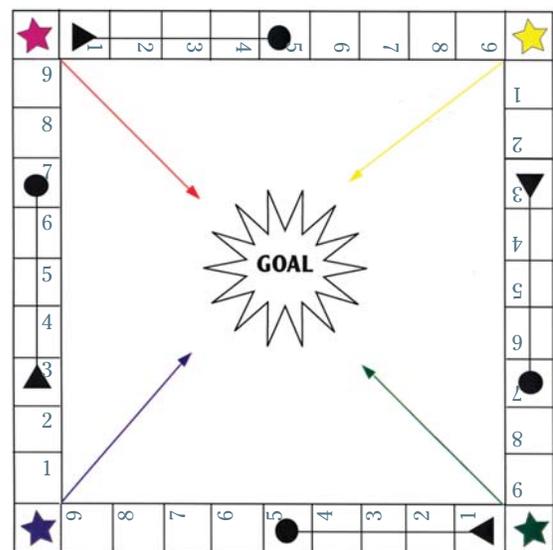
Three or four kindergartners can play the modified board game. The materials include a homemade game board (right), eight markers (two each of four different colors—for example, red, blue, yellow, and green) that coordinate with colors on the board, and 20 numeral cards (four each of numbers 1–5). The object of the game is to be the first player to get both markers in the Goal (see “Kindergarten Game Rules”).

Kindergarten Game Rules

- ▲ Each player takes two markers of the same color and places them on the board on the star of that color.
- ▲ The cards are shuffled and placed face down in a stack on the board near the Goal.
- ▲ The players decide who will go first, second, and third.
- ▲ On her turn, a player turns over the top card of the stack to show the number on the face of the card. She has a choice between advancing a marker by the number that came up or by double the number. For example, if a 3 comes up, the player can move one marker either 3 or 6 steps.
- ▲ When a marker lands on a triangle, it can slide four spaces to the circle at the end of the line connecting the two shapes.
- ▲ When a marker lands on a star, it can jump 10 squares to the next star.
- ▲ If a marker lands on a space already occupied by another player’s marker, the marker already there is sent back to the nearest star.
- ▲ To go into the Goal, a marker must land exactly on its starting point—the star of the same color. [Let children negotiate the rule about what happens when the exact number does not come up. Negotiations are good not only for children’s sociomoral development but also for their development of logic.]
- ▲ When a marker has gone around the board clockwise and returns to its starting point, the player is ready to move it into the Goal. The first person to get both markers in the Goal is the winner.

To understand the reasons for the game modifications for kindergartners, it is important to understand what Piaget meant by logico-mathematical knowledge. He made a fundamental distinction among three kinds of knowledge according to their ultimate sources—physical, social-conventional, and logico-mathematical knowledge (Piaget [1945] 1962, [1967] 1971).

Physical knowledge is knowledge of objects in the external world, such as their color and weight. The knowledge that a marble rolls away but a die comes to rest is also an example of physical knowledge. Examples of *social-conventional knowledge* are knowledge of holidays such as Thanksgiving and knowledge of written and spoken languages. Holidays and languages were created by convention among people.



Logico-mathematical knowledge

While the sources of physical and social-conventional knowledge are external to the individual, *logico-mathematical knowledge* consists of mental relationships—relationships originating in each person’s mind. If we see a red marker and a blue one, for example, we can think about them as being *different* from one another because of their colors or think of them as *similar* because of their shapes. It is just as true to say that they are different as it is to say that they are similar, because mental relationships are made by the person who puts the objects into relationships. If a person thinks about the markers as being different, the two objects are different for that person. For the person who ignores the colors of the markers, the markers are similar. A third example of a mental relationship a person can create between the markers is the numerical relationship *two*.

Classification. There are many kinds of logico-mathematical relationships an individual can create. The diagram titled “Knowledge Fostered by the Kindergarten Game” shows some of them. The first one is *classification* (Inhelder & Piaget [1959] 1964). The Japanese teachers introduced a classificatory relationship when they modified the game to offer two categories of use for the number on a card: (a) advancing a game piece by the number that came up or (b) advancing it by double the number.

Seriation. The second kind of logico-mathematical relationship shown in the diagram is *seriation* (Inhelder & Piaget [1959] 1964)—the ordering of objects according to relative differences. An example is the ordering of players according to who is winning, who is next, and who is last. Another example is the ordering of possible moves according to advantages—(1) ordinary moves that do not offer any additional advantage, (2) moves that permit a slide (4 extra squares), and (3) moves that permit a jump (10 extra squares).

Numerical relationships. The third kind of logico-mathematical relationship in the diagram is *numerical relationships* (Piaget & Szeminska [1941] 1965), which players make constantly while playing the kindergarten board game. For example, if a player turns over a 3, he or she can advance 3 spaces or $3 + 3$ spaces.

Spatial relationships. Fourth in the diagram is *spatial relationships* (Piaget & Inhelder [1948] 1967; Piaget, Inhelder, & Szeminska [1948] 1960). Getting closer to the Goal by sliding to the next circle or jumping to the next star is an example of a spatial relationship that is closely related to numerical relationships.

Temporal relationships. The last kind of logico-mathematical relationship shown is *temporal relationships* (Piaget [1946] 1969). The rule stating that the first person to place both markers in the Goal is the winner involves temporal, spatial, and numerical relationships.

There are other kinds of logico-mathematical relationships, but these examples are enough to explain how the changes the teachers made in the board game encouraged five- and six-year-olds to think logico-mathematically.

Modifying the game

The changes include modifications to the board, to the numbers that can come up, to the number of markers each player uses, and to what happens at the end of a move.

Changes to the board. The total number of squares was reduced from 60 to 40. The reduction makes the game shorter, and a shorter game keeps young children more alert. Logico-mathematical knowledge develops when children are mentally active.

The four individual goals were replaced by one common goal area because separate goals hindered the making of comparisons. Teachers heard comments like this after introducing one common Goal: “You finally got one in here. I’ve had one in here for a long time.” This comment reflects the temporal, spatial, and numerical relationships a child made. The child was saying that she took less time to cover the same distance.

Stars were introduced in each of the four corners, and children could now send opponents back to the nearest star. A child whose marker is sent all the way back to Start in Sorry! is unduly discouraged, and emotional upsets prevent children from reasoning logico-mathematically.

Changes in the numbers that can come up. In our first grade version of Sorry! the numbers on the die ranged from 0 to 9 or from 1 to 10. For kindergartners, the range was reduced to 1–5 because doubles up to $5 + 5$ are much easier to remember than doubles of 6 or more.

A change in what to do with the number that came up. Two categories were introduced so that children could compare the advantages of advancing a marker by the number that came up or by double the number.

A change in the number of markers. The number of markers for each player was reduced from four to two because thinking about four markers is impossible for kindergartners. In fact, many five- and six-year-olds can think only about one marker at a time.

A general trend in the development of logico-mathematical knowledge is the ability to put more and more

Knowledge Fostered by the Kindergarten Game							
	Physical knowledge	Social-conventional knowledge	Logico-mathematical knowledge				
			Logico-arithmetical knowledge		Spatio-temporal knowledge		
			Classification	Seriation	Numerical relationships	Spatial relationships	Temporal relationships
Kindergarten game			X	X	X	X	X

elements into a single, simultaneous relationship. In the seven examples that follow, the children in examples 1–3 thought only about one marker, but the more advanced children in examples 4–7 thought about both markers.

Changes in what happens at the end of a move. The teachers provided for four possible outcomes at the end of a move so that children would be encouraged to compare their advantages:

- landing on a space and doing nothing else
- landing on a triangle and sliding to a circle
- landing on a star and jumping to the next star
- landing on a space occupied by an opponent's marker and sending that marker back to the closest star.

In summary, the teachers decreased the difficulty of the arithmetic and increased the other logico-mathematical relationships children could make. Children could now advance by the number that came up on a card or by double the number (two possibilities). They could consider moving one marker or the other ($2 \times 2 = 4$ possibilities). Also, depending on which square a marker landed on, there were four possible outcomes at the end of a move—staying put, sliding to a circle, jumping to the next star, or sending an opponent back to the nearest star ($2 \times 2 \times 4 = 16$ possibilities).

Constructing relationships, developing thinking

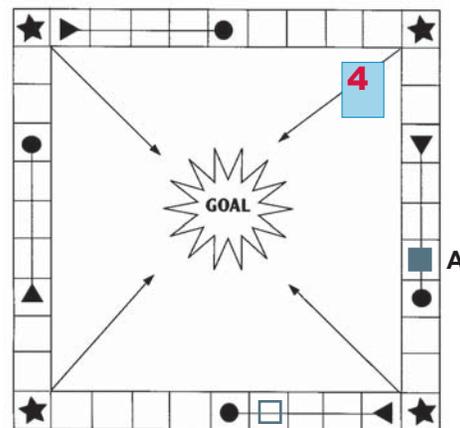
The following examples of seven children's plays illustrate how logico-mathematical relationships do not exist in the game itself; they must be *constructed* by each child. A game may offer 16 possibilities for the child to consider, but many of them do not exist for children who cannot think about them.

1. Risa always moved the same marker by the number that came up on a card. When Risa turned over a card on her turn, she always moved the same marker by the number on the card. She could state the rules of the game as well as all the doubles, such as $3 + 3 = 6$, $4 + 4 = 8$, and $5 + 5 = 10$. When she took her turn, however, the advantage of doubling the number or of advancing the other marker never seemed to occur to her. She thought only about the number that came up and did not put it into a relationship with any other factor. Furthermore, like most young children, Risa concentrated on one marker and did not consider the second marker till the first completed its circuit.

2. Kazuki always moved the same marker by double the number that came up. For example, when a 4 came up, Kazuki moved marker A by 8 steps (see "Kazuki's Move"), not noticing that it would have been more advantageous to move it by 4 steps, thereby landing that marker on a star and jumping it forward to the next star.

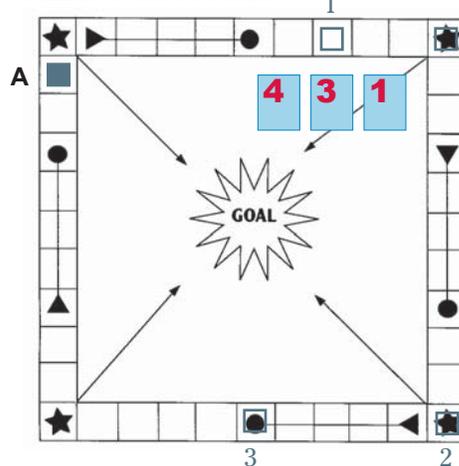
Kazuki's logico-mathematical thinking was more advanced than Risa's in that he had figured out that doubling the number would get him to the Goal faster. However, he focused exclusively on doubling each number and did not think about the possibility of a jump or a slide. These possibilities did not exist for him because he could not *think* about them.

Kazuki's Move



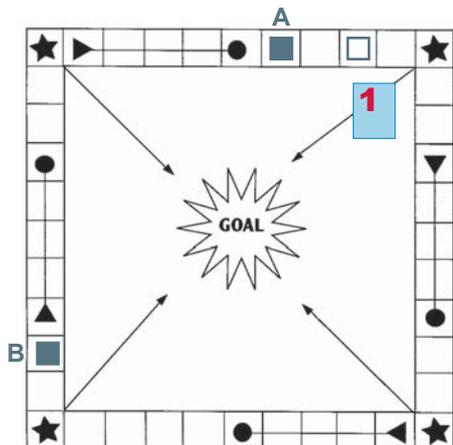
3. Takuma considered whether to move a marker by the number that came up or by its double. Takuma turned over a 4 and advanced marker A by 8 steps (designated by the number 1 in "Takuma's Three Turns"), thinking that this would take him closer to the Goal than advancing by only 4 steps. On his second turn a 3 came up, and he did not double this number because he wanted to land on a star and jump to the next star (see 2 in "Takuma's Three Turns"). On his third turn a 1 came up, and he did not double this number because he saw the possibility of landing on a triangle and sliding to a circle (see 3). On each turn, Takuma thus put the numerical differences into relationships with the possibility of advancing even further with a jump or a slide.

Takuma's Three Turns

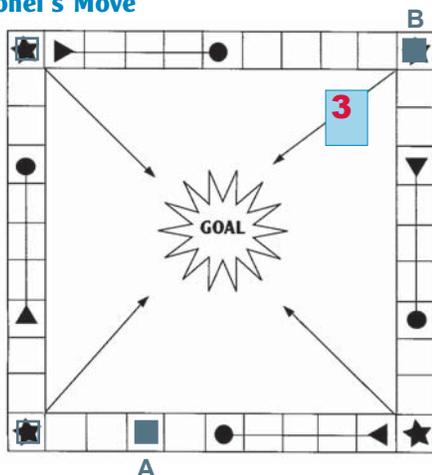


Takuma illustrates the interrelationships among the mental relationships listed in the diagram about logico-mathematical knowledge. When a child makes progress in *numerical thinking*, he also makes progress in *spatio-temporal thinking* about how to advance further spatially in the same amount of time.

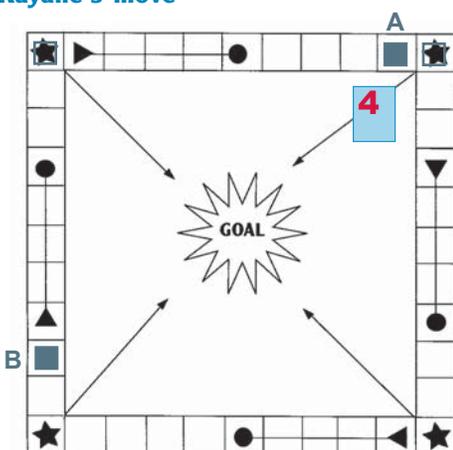
Takuma's Move, Next Day



Kohei's Move



Kayane's move



6. Yoshiki considered the four possibilities described in example 5 and sent back another player's marker. Yoshiki turned over a 3 and thought about the possibility of moving A by 3 or 6 steps (see "Yoshiki's Move," top right). He then thought about the possibility of moving B by 3 or 6 steps,

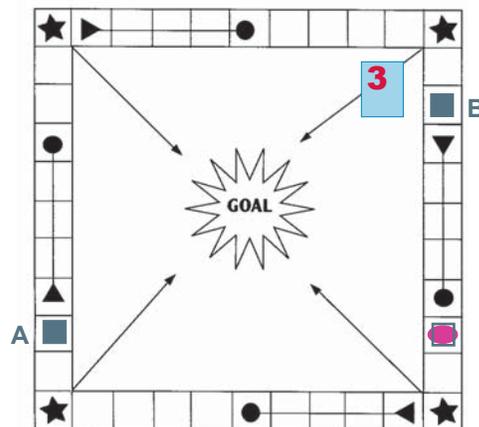
We can see that Takuma considered more possibilities than Risa or Kazuki did. On the next day, however, he turned over a 1 and doubled it to move marker A, which he had been advancing (see left, "Takuma's Move, Next Day"). He was thinking only about getting marker A to the Goal and did not notice the possibility of a slide by moving marker B by one step. Like Kazuki and Risa, Takuma focused on only one marker at a time.

4. Kohei considered two possibilities—moving marker A by the number that came up or marker B by the number that came up. Kohei turned over a 3 when his markers A and B were in the positions shown in "Kohei's Move" (left). He considered the possibility of moving A by 3 steps to land on a star and jump to the next star. He then considered the possibility of moving B by 3 steps to land on a triangle and slide to the circle. He decided to move A and jump to the next star.

5. Kayane considered four possibilities—moving marker A by the number that came up or by its double, or moving marker B by the number or its double. Kayane turned over a 4 (left), and she methodically compared the desirability of each of the following four possibilities:

- moving marker A by 4 steps and sliding to the circle
 - moving marker A by 8 steps and landing on the same circle
 - moving marker B by 4 steps and advancing only 4 steps
 - moving marker B by 8 steps and jumping to the next star
- Kayane decided to move B by 8 steps.

Yoshiki's Move

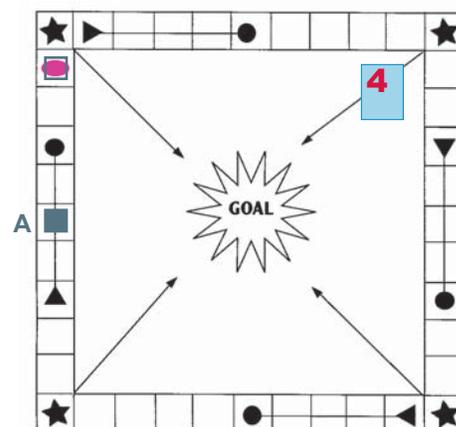


decided on the latter, and exclaimed "Sorry!" as he sent an opponent's marker (the oval) back to a star.

Although Yoshiki sent back another player's marker, he cannot be said to be at a higher level of logico-mathematical reasoning than Kayane (example 5), because his move did not involve a strategy beyond comparing the four possibilities.

7. Umi considered the four possibilities described in example 5 and compared the relative advantages of advancing her own marker as far as possible versus sending back another player's marker. When Umi turned over a 4 and exclaimed "Eight!" her marker A was in the position shown in "Umi's Move" (below). She hesitated, thought about the fact that an opponent was

Umi's Move



within one space of reaching the Goal, and decided to move marker A by 4 spaces to send the opponent's marker back to a star. Umi compared the advantage of advancing her marker by 8 steps with that of advancing it only 4 steps but sending back an opponent 9 steps.

The new kindergarten board game permitted seven children at very different levels of logico-mathematical knowledge to play together. Risa was at the lowest level; the only relationship she made was between the number that came up on a card and the number of steps to move her marker—always the same marker. By contrast, Kazuki had figured out that doubling the number got him closer to the Goal; thus he automatically doubled the number that came up. However, like Risa, he always moved the same marker. Takuma was more advanced in that he thought about both possible moves and related each to the possibility of a jump or a slide. His thinking was more advanced than that of Risa or Kazuki, but he too thought only about one marker and did not notice the possibility of sliding ahead by moving his other marker. Kohei thought about both markers but not about doubling the number.

By contrast, Kayane, Yoshiki, and Umi all thought about both markers and the relative advantages of moving each marker by the number that came up or by double that number.

The teacher's role

It is clear from the preceding discussion that an important part of the teacher's role is to provide appropriate materials and plenty of time to play good games. Children may or may not benefit from all the possibilities a game offers, but if a game is too hard, too easy, or inherently flawed (see "Go Fish and Making Families"), it does not help children develop logico-mathematically.

Creating the right atmosphere

Before thinking about appropriate games, the teacher's first task is to create a sociomoral atmosphere in the classroom so children can concentrate on playing the games. If they focus on bickering and fighting, the children need to learn how to negotiate solutions.

For example, if a child comes to the teacher complaining that So-and-So always insists on going first, the teacher can ask, "What do you think you can do about this problem?" rather than solving the problem for the child. If the child has no idea what to do, another question may be in order, such as "Have you told So-and-So that it's not fair if other people don't get a chance to go first?" (The answer is usually no.) If questions like these do not help, the teacher may need to make another suggestion, like "Have you thought about making a rule to suggest to your group so that everybody will have a chance to go first?"

Go Fish and Making Families— A Tale of Two Games

The popular card game Go Fish has great appeal for kindergarten-age children; however, it does little for their development of logic because the unknown face-down cards in the "pond" encourage random guessing.

Go Fish. The Go Fish deck contains a total of 32 cards having four identical pictures of eight kinds of fish. When three or four children play, five cards are dealt to each player. The rest of the cards are spread out, face down. They constitute the pond. The object of the game is for players to make sets of four identical cards by asking others for specific cards, one at a time. The person who makes the most sets is the winner.

For example, John may begin by asking Suzie, "Do you have a snapper?" If Suzie has one, she has to give it to John. John continues asking for cards until he is told, "I don't have any. Go fish." John then takes a card from the pond, and the turn to ask for cards passes to the person who said, "I don't have any."

A modification of Go Fish is called Making Families. Because this game has no pond, it offers children opportunities to engage in logical reasoning instead of guessing.

Making Families. Like Go Fish, the object of this game is to make the most sets of four matching cards. The deck contains only five sets of four identical cards (a total of 20 cards). In Making Families (described in detail in Kamii and DeVries [1980]) all the cards are dealt to the three or four players; thus there is no pond. If three children are playing, for example, and Suzie says to John that she does not have any snappers, John can deduce that Kathy must have one. If John already has two snapper cards and Suzie does not have any, John can deduce that Kathy must have two snappers.

The fact that there are only 20 cards instead of 32 cards to consider also helps young children think logically and clearly. Having four players complicates the game, but kindergartners become impressively adept at deducing exactly where to find a card.

An important principle to keep in mind is that children need to make their own decisions and try to solve their own problems. The adult can make suggestions, but it is up to the children to decide whether or not to accept the teacher's suggestions. If the teacher makes decisions for children, children cannot assume responsibility for solving their own problems.

More specific ways of handling conflicts can be found in DeVries and Zan (1994). Suffice it to say here that children's logic develops enormously when they negotiate solutions with other children. Comparing the desirability of rule A over rule B requires logico-mathematical thinking, and figuring out how to convince another player requires even more thinking.

Joining the game

When children become able to play games independently, the best thing for the teacher to do is sit down and play the games with the children. It is important for the teacher to remember that the logico-mathematical relationships children develop come neither from the materials nor from the teacher. Children develop their logico-mathematical knowledge by using the mental relationships they have already made.

In the earlier examples, Kazuki used the relationships he had made when he was at Risa's level. Takuma used the relationships he had made when he was at Kazuki's level. The other examples are not in precise developmental order, but Kayane was clearly using the relationships that could be observed in all the examples that preceded. Umi put more elements into relationship than all the others described.

We learn the most about children's levels of thinking when we play with them, and this assessment tells us how best to intervene. A teacher playing with Risa might think aloud, saying, "Hmm . . . I could take 2 steps or 4 steps. What should I do?" Risa may or may not get the hint from this remark, but a teacher who understands Risa's level would know that this indirect suggestion is probably at the right level. With a child who is at a higher level, like Takuma, the teacher might think aloud by considering the two possibilities he thought about and then say, "I wonder if there is anything else I could think about."

In conclusion

Children's ability to think logico-mathematically is important to develop because this ability is necessary in all the subjects, especially science and mathematics. The relationships shown in the diagram earlier (p. 22) are still undifferentiated in kindergarten, but they become differentiated as separate systems between the ages of 6 and 10 as children group classificatory rela-

tionships together, seriation relationships together (Inhelder & Piaget [1959] 1964), numerical relationships together (Piaget & Szeminska [1941] 1965), spatial relationships together (Piaget, Inhelder, & Szeminska [1948] 1960; Piaget & Inhelder [1948] 1967), and temporal relationships together (Piaget [1946] 1969).

An example of the gradual differentiation of systems can be seen in the conservation-of-number task. Many four- and five-year-olds think there are more counters in the longer row below because the longer row occupies more space.



The reason for this judgment is that most four-year-olds do not yet have number in their heads. Because they therefore cannot put the objects into numerical relationships, the only quantitative judgment they can make is based on the space occupied by the objects. As they group numerical relationships together, numerical thinking becomes separated from spatial thinking and children begin to say that the two rows have the same number.

Kindergarten-age children develop many kinds of relationships in an interrelated way, as we saw in the examples of the seven children. Teachers can encourage the development of this undifferentiated network so children will go on to build a numerical system that will lead to algebra and a spatial system that will lead to geometry.

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